

Some properties of the newly observed $X(1835)$ state at BES

Xiao-Gang He^{1,2}, Xue-Qian Li¹, Xiang Liu^{1,a}, Jian-Ping Ma³

¹ Department of Physics, Nankai University, Tianjin, P.R. China

² NCTS/TPE, Department of Physics, National Taiwan University, Taipei, Taiwan, R.O.C.

³ Institute of Theoretical Physics, Academia Sinica, Beijing, P.R. China

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Abstract. Recently the BES collaboration has announced the observation of a resonant state in the $\pi^+\pi^-\eta'$ spectrum in $J/\psi \rightarrow \gamma\pi^+\pi^-\eta'$ decay. Fitting the data with a 0^{-+} state, the mass is determined to be 1833.7 MeV with 7.7σ statistic significance. This state is consistent with the one extracted from previously reported $p\bar{p}$ threshold enhancement data in $J/\psi \rightarrow \gamma p\bar{p}$. We study the properties of this state using QCD anomaly and QCD sum rules, assuming $X(1835)$ to be a pseudoscalar, and we show that it is consistent with the data. We find that this state has a sizeable matrix element $\langle 0|G\tilde{G}|G_p\rangle$, leading to branching ratios of $(2.61-7.37) \times 10^{-3}$ and $(2.21-10.61) \times 10^{-2}$ for $J/\psi \rightarrow \gamma G_p$ and for $G_p \rightarrow \pi^+\pi^-\eta'$, respectively. Combining the calculated branching ratio of $J/\psi \rightarrow \gamma G_p$ and data on threshold enhancement in $J/\psi \rightarrow \gamma p\bar{p}$, we determine the coupling for the $G_p-p\bar{p}$ interaction. We finally study the branching ratios of the other $J/\psi \rightarrow \gamma +$ three mesons decay modes. We find that $J/\psi \rightarrow \gamma G_p \rightarrow \gamma(\pi^+\pi^-\eta, KK\pi^0)$ can provide useful tests for the mechanism proposed.

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Recently the BES collaboration has announced the observation of a resonant state in the $\pi^+\pi^-\eta'$ spectrum [1] in $J/\psi \rightarrow \gamma\pi^+\pi^-\eta'$. A fit for a 0^{-+} resonant state with a Breit-Wigner function yields a mass $M = 1833.7 \pm 6.5(\text{stat.}) \pm 2.7(\text{syst.})$ MeV, a width $\Gamma = 67.7 \pm 20.3(\text{stat.}) \pm 7.7(\text{syst.})$ MeV and a production branching ratio $BR(J/\psi \rightarrow \gamma X)BR(X \rightarrow \pi^+\pi^-\eta') = (2.2 \pm 0.4(\text{stat.}) \pm 0.4(\text{syst.})) \times 10^{-4}$ with a 7.7σ statistic significance. The mass of this state is consistent with that extracted from the enhanced threshold $p\bar{p}$ events in [2] $J/\psi \rightarrow \gamma p\bar{p}$. The properties of this state cannot be explained by known particles. Various models have been proposed to explain the possible resonance [3–14]. Further experimental confirmation of this state is needed.

There are some hypothetic candidate particles which may fit in the picture. Some of the possibilities include a $p\bar{p}$ bound state [3–13], and a pseudoscalar glueball state [3, 14]. The existence of glueballs is a natural prediction of QCD. The prediction for the glueball masses is, however, a non-trivial task. QCD sum rules [15–18] and lattice QCD [19, 20] calculations obtain the result that the lowest pseudoscalar glueball mass is in the range of 1800 to 2600 MeV with lattice calculations giving a mass in the upper range. One cannot rule out the possibility that the resonant state $X(1835)$ is a glueball based on our present understanding of the glueball masses alone. At this stage

there are no compelling reasons to believe that the resonance is a $p\bar{p}$ bound state either [4–13]. Of these speculative particle states, although they are attractive, the existences are far from having been established. More theoretical and experimental efforts are needed to go further. At a more modest level, even to know whether the data from BES can be consistently explained by a specific resonance and to further test the mechanism, more information on the properties of the resonance is needed, such as how it is produced in radiative J/ψ decays and how it decays into other particles.

In this work we study the properties of the $X(1835)$ resonance using QCD anomaly and QCD sum rules, assuming that this state is a pseudoscalar G_p which couples strongly with two gluons which may or may not be a glueball or a $p\bar{p}$ bound state depending on whether this state has large mixing. We find that the matrix element $\langle 0|G\tilde{G}|G_p\rangle$ is larger than $\langle 0|G\tilde{G}|\eta(\eta')\rangle$, indicating a large glue content in G_p , which is usually referred to as a glueball in the literature. This leads to the large branching ratios of $(2.61-7.37) \times 10^{-3}$ for $J/\psi \rightarrow \gamma G_p$ and $(2.21-10.61) \times 10^{-2}$ for $G_p \rightarrow \pi^+\pi^-\eta'$. The coupling for the $G_p-p\bar{p}$ interaction can also be determined. We finally discuss how other G_p decay modes can be used to test the mechanism.

There has been published a considerable amount of literature on the production of a pseudoscalar in radiative J/ψ decays, in terms of QCD sum rules [21] and perturbative QCD calculations [22–26]. We follow the QCD sum

^a e-mail: liuxiang726@mail.nankai.edu.cn

rule approach in [21], such that we can treat radiative J/ψ decays into η , η' and G_p in the same framework with QCD anomaly. In this framework, the radiative J/ψ decay amplitudes are determined as follows: one first evaluates the internal charm quark loop contribution to the interaction of two photons \rightarrow two gluons, and then saturates the $c\bar{c}$ pair which couples to one of the external photons by J/ψ and other resonant states using the standard procedure of the QCD sum rules. The two gluons are then converted into the related pseudoscalar states. This approach works best when the final pseudoscalar has a mass-squared much smaller than $4m_c^2 \sim m_{J/\psi}^2$. For a pseudoscalar of mass 1833.7 MeV, there may be large corrections from significant two photon to multi-gluon couplings since the factor $m_{G_p}^2/m_{J/\psi}^2$ may not be sufficient to suppress higher order contributions. However, one expects that the matrix elements of operators converting multi-gluon to a pseudoscalar G_p must be smaller compared with that from the leading two gluon operator. The situation may not be too severe to damage the whole picture of the two gluon scenario. Also, in our later discussions we will only use the ratios of two different $J/\psi \rightarrow \gamma X_i$ branching fractions, and a large part of the uncertainty is expected to be cancelled. One expects that the error range can be controlled to within a factor of two.

In this calculation the two gluon operator with appropriate quantum numbers is $G_{\mu\nu}\tilde{G}^{\mu\nu}$. The matrix elements converting the two gluons into a pseudoscalar X_i is usually parameterized as $f_i m_i^2 = \langle 0 | (3\alpha_s/4\pi) G_{\mu\nu}\tilde{G}^{\mu\nu} | X_i \rangle$. Since the rest of the decay amplitude for $J/\psi \rightarrow \gamma X_i$ is independent of the final pseudoscalar state, the ratio of radiative branching fractions for the X_i and X_j states is simply given by [21]

$$R_{ij} = \frac{BR(J/\psi \rightarrow \gamma X_i)}{BR(J/\psi \rightarrow \gamma X_j)} = \frac{|f_i m_i^2|^2 (1 - m_i^2/m_{J/\psi}^2)^3}{|f_j m_j^2|^2 (1 - m_j^2/m_{J/\psi}^2)^3}. \quad (1)$$

The parameters f_{η,η',G_p} play a crucial role in the determination of $J/\psi \rightarrow \gamma G_p$ in the QCD sum rule approach. The parameters $f_{\eta,\eta'}$ can easily be obtained from the QCD anomaly relations in the limit that the strange quark mass is much larger than the up and down quark masses. One has [27–29]

$$\begin{aligned} \left\langle 0 \left| \frac{3\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} \right| \eta \right\rangle &= \sqrt{\frac{3}{2}} (\cos\theta f_8 - \sqrt{2}\sin\theta f_0) m_\eta^2, \\ \left\langle 0 \left| \frac{3\alpha_s}{4\pi} G_{\mu\nu}\tilde{G}^{\mu\nu} \right| \eta' \right\rangle &= \sqrt{\frac{3}{2}} (\sin\theta f_8 + \sqrt{2}\cos\theta f_0) m_{\eta'}^2, \end{aligned} \quad (2)$$

where θ is the η - η' mixing angle with $\eta = \eta_8 \cos\theta - \eta_0 \sin\theta$ and $\eta' = \eta_8 \sin\theta + \eta_0 \cos\theta$. $f_{8,0}$ are the decay constants of the $SU(3)$ octet η_8 and the singlet η_0 .

There are many theoretical studies on the values of θ and $f_{8,0}$. In our study since only η , η' and J/ψ are involved, we will use related processes to determine $f_{8,0}$ and θ . These processes include $\eta \rightarrow \gamma\gamma$, $\eta' \rightarrow \gamma\gamma$ and $R_{\eta'\eta} = BR(J/\psi \rightarrow$

$\gamma\eta')/BR(J/\psi \rightarrow \gamma\eta)$. We have

$$\begin{aligned} \Gamma(\eta \rightarrow \gamma\gamma) &= \frac{m_\eta^3}{96\pi^3} \alpha_{\text{em}}^3 \left(\frac{\cos\theta}{f_8} - 2\sqrt{2}\frac{\sin\theta}{f_0} \right)^2, \\ \Gamma(\eta' \rightarrow \gamma\gamma) &= \frac{m_{\eta'}^3}{96\pi^3} \alpha_{\text{em}}^3 \left(\frac{\sin\theta}{f_8} + 2\sqrt{2}\frac{\cos\theta}{f_0} \right)^2, \\ R_{\eta'\eta} &= \left| \frac{(\sin\theta f_8 + \sqrt{2}\cos\theta f_0) m_{\eta'}^2}{(\cos\theta f_8 - \sqrt{2}\sin\theta f_0) m_\eta^2} \right|^2 \\ &\quad \times \frac{\left(1 - m_{\eta'}^2/m_{J/\psi}^2\right)^3}{\left(1 - m_\eta^2/m_{J/\psi}^2\right)^3}. \end{aligned} \quad (3)$$

Using the experimental values $BR(\eta \rightarrow \gamma\gamma) = (39.43 \pm 0.26)\%$, $BR(\eta' \rightarrow \gamma\gamma) = (2.12 \pm 0.14)\%$, $BR(J/\psi \rightarrow \gamma\eta) = (8.6 \pm 0.8) \times 10^{-4}$ and $BR(J/\psi \rightarrow \gamma\eta') = (4.31 \pm 0.30) \times 10^{-3}$ [30], we obtain the ranges (central values) for the parameters as follows: $\theta = (-16.88^\circ) - (-18.60^\circ) (-17.72^\circ)$, $f_8 = 0.98f_\pi - 1.04f_\pi$ ($1.01f_\pi$) and $f_0 = 1.06f_\pi - 1.21f_\pi$ ($1.08f_\pi$) with $f_\pi = 132$ MeV being the pion decay constant. The correlations of these parameters are shown in Fig. 1. These values are consistent with the values determined from other considerations [27, 31] for θ and $f_{8,0}$. This gives us some confidence in using the QCD sum rule results for $J/\psi \rightarrow \gamma\eta$, $J/\psi \rightarrow \gamma\eta'$, and as well as for $J/\psi \rightarrow \gamma G_p$. We will use the above values for θ and $f_{8,0}$ in our later discussions.

So far the parameter f_{G_p} is not well understood. To obtain some information, we use the QCD sum rules to calculate it. The basic idea of the QCD sum rule analysis in the present case is to match the dispersion relation involving the hadronic spectral density $\varrho(s)$ to the vacuum topological susceptibility $T(-q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T[j_{\text{ps}}(x) j_{\text{ps}}(0)] | 0 \rangle$ with the result found by using the operator product expansion. We follow [17, 18] by making a Borel transformation on $T(s)$ with $\int_0^\infty \text{Im}(T(s)) e^{-s/M^2} ds/s$ and $\int_0^\infty \text{Im}(T(s)) e^{-s/M^2} ds$, and including the two ground

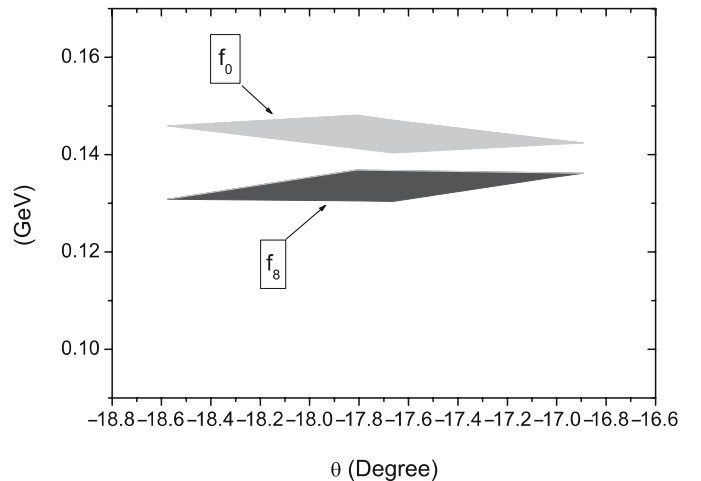


Fig. 1. The dependence of f_0 and f_8 on θ . The ranges are due to 1σ errors of the data points

pseudoscalar states η and η' , and G_p in the resonant spectral density, to obtain the leading order matching conditions [17, 18]

$$\begin{aligned} & f_{G_p}^2 m_{G_p}^2 e^{-m_{G_p}^2/M^2} + f_\eta^2 m_\eta^2 e^{-m_\eta^2/M^2} + f_{\eta'}^2 m_{\eta'}^2 e^{-m_{\eta'}^2/M^2} \\ &= \int_0^{s_1} b s e^{-s/M^2} ds + \tilde{D}_4 + O\left(\frac{1}{M^2}\right) + \text{inst.}, \\ & f_{G_p}^2 m_{G_p}^4 e^{-m_{G_p}^2/M^2} + f_\eta^2 m_\eta^4 e^{-m_\eta^2/M^2} + f_{\eta'}^2 m_{\eta'}^4 e^{-m_{\eta'}^2/M^2} \\ &= \int_0^{s_1} b s^2 e^{-s/M^2} ds - \tilde{D}_6 + O\left(\frac{1}{M^2}\right) + \text{inst.}, \quad (4) \end{aligned}$$

where ‘‘inst.’’ indicates direct instanton effects [32]. $\tilde{D}_{4,6}$ are related to the gluon condensations, $D_4 = 4\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle$, and $D_6 = 8g_s f_{abc}\langle 0|G_{\mu\alpha}^a G_\nu^{b,\alpha} G^{c,\nu\mu}|0\rangle$, with $\tilde{D}_4 = \pi(3\alpha_s/4\pi)^2 D_4$, $\tilde{D}_6 = \pi(3\alpha_s/4\pi)^2 D_6$. $b = (3\alpha_s/4\pi)^2 \times (2/\pi)(1 + 5\alpha_s/\pi)$. In our numerical evaluations, we will use $\langle 0|\alpha_s G^2|0\rangle = (7.1 \pm 0.9) \times 10^{-2} \text{ GeV}^4$ and the relation $\langle 0|g^3 f_{abc} G^a G^b G^c|0\rangle = 1.2 \text{ GeV}^2 \langle 0|\alpha_s G^2|0\rangle$ [33, 34]. We comment that there are other 0^{-+} states around the 1400 MeV region which may contribute to the spectrum density if these states contain a large two gluon contents. We will assume that the gluon contents are small in these states and that their contributions to the spectrum density can be neglected.

To determine f_{G_p} , we take the usual practice to find the parameters f_{G_p} and s_1 for a given Borel parameter M and look for a region where the dependence of f_{G_p} and s_1 on M is insensitive. We will neglect the direct instanton effect in our calculation and will come back to comment on the effects later. Note that the analysis with a fixed G_p mass here is different from previous ones [15–18, 33, 34], where the mass of G_p is taken as one of the parameters to be determined, and therefore the results are in general different. The solutions for f_{G_p} depend on the value of α_s , which we take to be the value at the scale $\mu = m_{G_p}$ with $\alpha_s = 0.35 \pm 0.05$. We find that solutions exist only for a restricted parameter space for α_s and $\tilde{D}_{4,6}$. In certain ranges, for a given set of input values of α_s and $\tilde{D}_{4,6}$, there are two solutions. For example with $\alpha_s = 0.39$, $\tilde{D}_4 = 1.99 \times 10^{-2} \text{ GeV}^4$ and $\tilde{D}_6 = 3.79 \times 10^{-3} \text{ GeV}^6$, we get the two solutions a) $s_1 = 3.2 \text{ GeV}^2$ and $f_{G_p} = 0.081 \text{ GeV}$, and b) $s_1 = 3.5 \text{ GeV}^2$ and $f_{G_p} = 0.091 \text{ GeV}$. When M is larger than 7 GeV, the solutions are fairly stable.

As long as we choose an M far above m_{G_p} , the power corrections from higher dimensional operators on the right hand sides of (4) can be suppressed. We note that for the solution with lower s_1 , the value of s_1 is smaller than $m_{G_p}^2$ which cannot be considered to be a good solution, since it implies that the continuum already starts to contribute to the sum rules in the resonant region in contradiction with the QCD sum rules assumptions. We therefore should choose the solution with the larger s_1 . This solution allows for a small gap between the resonances and the continuum. We show the results in Fig. 2, allowing α_s to vary from 0.3626 (where a solution begins to exist) to 0.4 (the 1σ allowed upper bound), and all other quantities, $\tilde{D}_{4,6}$, $f_{\eta,\eta'}$ and θ to vary within 1σ error ranges. We see that

the dependence on M is very mild. We conclude that there are consistent solutions from QCD sum rules for a pseudoscalar of mass 1833.7 MeV, and we obtain a conservative range for f_{G_p} with

$$f_{G_p} = 0.072\text{--}0.100 \text{ GeV}. \quad (5)$$

When we obtained the range of f_{G_p} , the direct instanton effects were neglected. At large Q , the instanton effects are suppressed [14–16, 35], for example a spike distribution for the instanton density results in an exponential suppression when Q^2 becomes larger than a GeV^2 or so [15, 16]. With the glueball mass fixed at 1835 MeV, the suppression may not be sufficient to neglect the contributions from the direct instanton. The effects of a direct instanton may be substantial. However the detailed calculations depend on the instanton density and the density shape. A reliable evaluation of the instanton effects is difficult. Nevertheless, model calculations show that instanton effects may be important due to a modification to the normalization of the

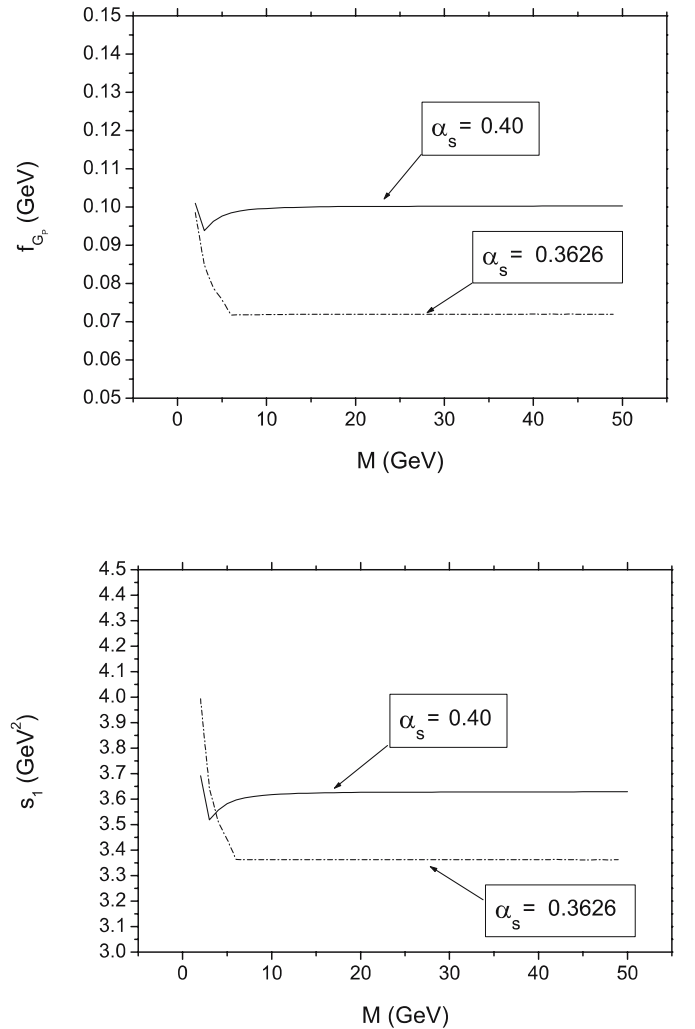


Fig. 2. f_{G_p} and s_1 as functions of M . The upper and lower bounds are for 1σ ranges of $\tilde{D}_{4,6}$, $f_{\eta,\eta'}$ and θ with $\alpha_s = 0.4$ and $\alpha_s = 0.3626$, respectively

Wilson coefficients for the relevant operators. We will not go into the specific details, as it is too model dependent. Some detailed discussions for direct instanton effects can be found in [15, 16]. We emphasize that the QCD sum rule results should be taken as an estimate within a factor of two. In our later discussions we will use the range for f_{G_p} obtained above as a reference. Should a more precise value be determined with some method, one can easily rescale the values accordingly.

With the above range for f_{G_p} , we find that the matrix element $\langle 0 | \alpha_s G \tilde{G} | G_p \rangle$ is larger than $\langle 0 | \alpha_s G \tilde{G} | \eta(\eta') \rangle$, indicating that G_p contains a large gluon content. With f_{G_p} determined, we are now able to obtain information on the range for $BR(J/\psi \rightarrow \gamma G_p)$, combining (1) and experimental data on $J/\psi \rightarrow \gamma \eta(\eta')$. We have

$$BR(J/\psi \rightarrow \gamma G_p) = (2.61-7.37) \times 10^{-3}. \quad (6)$$

Using the BES data $BR(J/\psi \rightarrow \gamma G_p) BR(G_p \rightarrow \pi^+ \pi^- \eta') = (2.2 \pm 0.4(\text{stat.}) \pm 0.4(\text{syst.})) \times 10^{-4}$, we can therefore also obtain the branching ratio of G_p to $\pi^+ \pi^- \eta'$ using our estimate of $J/\psi \rightarrow \gamma G_p$. We have

$$BR(G_p \rightarrow \pi^+ \pi^- \eta') = (2.21-10.61) \times 10^{-2}. \quad (7)$$

The branching ratio for this decay is large, but it does not contradict known data.

If the enhanced threshold of $p\bar{p}$ events in $J/\psi \rightarrow \gamma p\bar{p}$ is also due to the same G_p state, we can obtain information about the interaction of G_p with a proton and anti-proton pair, $L = C_{GB} \bar{p} \gamma_5 p G_p$. Since the mass of G_p is slightly below the threshold of two proton masses, $2m_p$, one cannot simply take $BR(J/\psi \rightarrow \gamma G_p \rightarrow \gamma p\bar{p})$ to be equal to $BR(J/\psi \rightarrow \gamma G_p) BR(G_p \rightarrow p\bar{p})$. One must consider the off-shell effects of G_p in terms of the Breit–Wigner approach. We have

$$\begin{aligned} & BR(J/\psi \rightarrow \gamma G_p \rightarrow \gamma p\bar{p}) \\ &= C_{GB}^2 \frac{BR(J/\psi \rightarrow \gamma G_p)}{8\pi^2} \int_{4m_p^2}^{m_{J/\psi}^2} dq^2 \\ & \times \frac{(1 - q^2/m_{J/\psi}^2)^3 q^2}{(1 - m_{G_p}^2/m_{J/\psi}^2)^3} \frac{\sqrt{1 - 4m_p^2/q^2}}{(q^2 - m_{G_p}^2)^2 + \Gamma_{G_p}^2 m_{G_p}^2}. \end{aligned} \quad (8)$$

In the above, we have assumed that C_{GB} does not depend on q^2 sensitively and has been moved out from the integration sign.

Experimental data on $BR(J/\psi \rightarrow \gamma G_p \rightarrow \gamma p\bar{p}) = (7.0 \pm 0.4_{-0.8}^{+1.8}) \times 10^{-5}$ then implies

$$C_{GB} = 0.42-0.85. \quad (9)$$

One sees that threshold enhancement data in $J/\psi \rightarrow \gamma p\bar{p}$ can also be consistently explained.

We now discuss how the resonance G_p may decay into other particles. As has been pointed out earlier, this state has a large gluon contents; it may be a glueball which is a $SU(3)$ singlet. Of course one needs to be more open

minded in that it may has sizeable mixing with another state. We will take the state G_p to be a glueball state and study the consequences from the flavor symmetry point of view. One can easily study the branching ratios for $J/\psi \rightarrow \gamma G_p \rightarrow \gamma B\bar{B}$, with $B\bar{B}$ a pair of octet baryons. The mixing effect can easily be implemented by introducing some mixing parameters.

The coupling of a $SU(3)$ singlet G_p and octet baryon can be written as $L = C_{GB} G_p \text{Tr} B \gamma_5 B$ with $SU(3)$ flavor symmetry. In Table 1 we list the ratios of $r(B\bar{B}) = BR(J/\psi \rightarrow \gamma G_p \rightarrow \gamma B\bar{B})/BR(J/\psi \rightarrow \gamma G_p \rightarrow \gamma p\bar{p})$ for possible decay modes. We comment that the $X(1835)$ contributions listed in Table 1 hold as long as the resonance is an $SU(3)$ singlet and does not depend on the size of C_{GB} . If the resonance transforms non-trivially under the flavor $SU(3)$ symmetry, the predictions would be different [36]. In principle experimental measurements of these branching ratios can provide important information about the nature of the resonance. However, the branching ratios for other baryon pair decay modes are much smaller than the branching ratio with a proton and anti-proton pair except the neutron and anti-neutron pair decay mode, which is then experimentally difficult to carry out. The usefulness of these decay modes depends on whether, near the resonance region, the resonance contributions dominate over the non-resonance continuum parts which we will comment on later.

A better test of the mechanism may come from other three meson decay modes of G_p . To have further information on the branching ratios for G_p decay into three meson modes, we here follow [37, 38] to use $U(3)$ chiral theory to describe how it couples to known meson particles. Notice that the use of $U(3)$ symmetry will not change our previous discussions on $G_p \rightarrow B\bar{B}$ results. The reason we use $U(3)$ chiral perturbation theory for the interaction is that it can naturally include many properties of chiral anomaly which our calculations for f_i depend on. To the leading order there are four terms which may cause G_p to decay [37, 38]:

$$\begin{aligned} L &= ia_1 \partial_\mu G_p \text{Tr} (\Sigma^\dagger \partial^\mu \Sigma \partial_\nu \Sigma^\dagger \partial^\nu \Sigma) \\ &+ ia_2 G_p \text{Tr} ((\Sigma^\dagger \partial^2 \Sigma - \partial^2 \Sigma^\dagger \Sigma) \partial_\nu \Sigma^\dagger \partial^\nu \Sigma) \\ &+ ia_3 G_p \text{Tr} (\chi \Sigma - \Sigma^\dagger \chi) \\ &+ ia_4 G_p \text{Tr} (\chi \Sigma^\dagger \partial_\nu \Sigma \partial^\nu \Sigma^\dagger - \chi \Sigma \partial_\nu \Sigma^\dagger \partial^\nu \Sigma), \end{aligned} \quad (10)$$

where $\Sigma = \exp[-i\sqrt{2}M/f]$ with $f = f_\pi/\sqrt{2}$ and M is the $U(3)$ meson nonet. χ is proportional to the light quark

Table 1. $r(B\bar{B})$ for different $J/\psi \rightarrow \gamma G_p \rightarrow \gamma B\bar{B}$ decays

$r(\Xi^0 \bar{\Xi}^0)$	1.58×10^{-3}
$r(\Xi^- \bar{\Xi}^-)$	1.38×10^{-3}
$r(\Sigma^+ \bar{\Sigma}^+)$	1.45×10^{-2}
$r(\Sigma^0 \bar{\Sigma}^0)$	1.38×10^{-2}
$r(\Sigma^- \bar{\Sigma}^-)$	1.28×10^{-2}
$r(\Lambda \bar{\Lambda})$	4.23×10^{-2}
$r(n\bar{n})$	0.96

masses and is given by $\chi = \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2)$. The last two terms in (10) come from explicit $U(3)$ (and $SU(3)$) breaking due to quark masses.

The two $SU(3)$ breaking terms in (10), if dominant, will lead to the main decay mode to be $G_p \rightarrow \pi KK$ [37, 38] with a very suppressed rate for $G_p \rightarrow \pi^+ \pi^- \eta'$. The BES data indicate that the decay mode $G_p \rightarrow \pi^+ \pi^- \eta'$ has a large branching ratio compared with other three particle decays (yet to be discovered); therefore, these two terms may be suppressed. If the first two terms dominate, we obtain the effective Lagrangian for the decay amplitude $G_p \rightarrow \pi^+ \pi^- \eta'$ to be

$$L = \frac{4}{\sqrt{3}f^3} (\sqrt{2} \cos \theta + \sin \theta) \times (a_1 \partial_\mu G_p (\partial^\mu \eta' \partial_\nu \pi^+ \partial^\nu \pi^- + \partial^\mu \pi^- \partial_\nu \pi^+ \partial^\nu \eta' + \partial^\mu \pi^+ \partial_\nu \eta' \partial^\nu \pi^-) + a_2 G_p (\partial^2 \eta' \partial_\nu \pi^+ \partial^\nu \pi^- + \partial^2 \pi^- \partial_\nu \pi^+ \partial^\nu \eta' + \partial^2 \pi^+ \partial_\nu \eta' \partial^\nu \pi^-)), \quad (11)$$

which leads to a decay amplitude,

$$M(G_p \rightarrow \pi^+ \pi^- \eta') = \frac{\sqrt{2} \cos \theta + \sin \theta}{\sqrt{3}f^3} \times (a_1 [(s - m_{G_p}^2 - m_{\eta'}^2)(s - 2m_\pi^2) + (t - m_{G_p}^2 - m_\pi^2)(t - m_{\eta'}^2 - m_\pi^2) + (u - m_{G_p}^2 - m_\pi^2)(u - m_{\eta'}^2 - m_\pi^2)] + 2a_2 [m_{\eta'}^2 (s - 2m_\pi^2) + m_\pi^2 (t - m_{\eta'}^2 - m_\pi^2) + m_\pi^2 (u - m_{\eta'}^2 - m_\pi^2)]), \quad (12)$$

where $s = (p_{\pi^+} + p_{\pi^-})^2$, $t = (p_{\pi^-} + p_{\eta'})^2$ and $u = (p_{\pi^+} + p_{\eta'})^2$.

To obtain the BES result for $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta'$ with the $BR(J/\psi \rightarrow \gamma G_p)$ obtained earlier, we have

$$|a_1|^2 - 0.376 \text{Re}(a_1 a_2^*) + 0.038 |a_2|^2 = (0.94 - 4.52) \times 10^{-5} \text{GeV}^{-2}. \quad (13)$$

Since there are two parameters, a_1 and a_2 , involved, with the data from $J/\psi \rightarrow \gamma G_p \rightarrow \gamma \pi^+ \pi^- \eta'$ alone it is not possible to predict branching ratios for other three meson decay modes. There are several kinematically allowed decay modes which can provide more information. These additional decay modes include $\eta' \pi^0 \pi^0$, $\eta \pi^+ \pi^-$, $\eta \pi^0 \pi^0$, $\eta K^+ K^-$, $\eta K^0 \bar{K}^0$, $\pi^0 K^0 \bar{K}^0$, $\pi^0 K^+ K^-$, $\pi^+ K^0 K^-$, $\pi^- \bar{K}^0 K^+$, and $\eta \eta \eta$. With the assumption that the dominant contributions come from the $a_{1,2}$ terms, even with

$SU(3)$ breaking effects from the final meson mass differences, one would have the following relations between the branching ratios: $BR(\pi^0 \pi^0 \eta'(\eta)) = BR(\pi^+ \pi^- \eta'(\eta))/2$, $BR(K^+ K^- \eta) = BR(K^0 \bar{K}^0 \eta)$, and $BR(\pi^0 K^+ K^-) = BR(\pi^0 K^0 \bar{K}^0) = BR(\pi^- K^+ \bar{K}^0)/2 = BR(\pi^+ K^- K^0)/2$. We list the ratios $r(p_1 p_2 p_3) = BR(G_p \rightarrow p_1 p_2 p_3) / BR(G_p \rightarrow \pi^+ \pi^- \eta')$ in Table 2. The branching ratio for $G_p \rightarrow \pi^+ \pi^- \eta$ can be larger than that for $G_p \rightarrow \pi^+ \pi^- \eta'$. No observation of $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta$ near the resonance would imply that there may be large cancellations between terms proportional to a_1 and a_2 for this decay mode. $G_p \rightarrow \pi KK$ are some other modes which may have large branching ratios near the resonance. The branching ratios here are different from those predicted if the resonance transforms non-trivially under the $SU(3)$ symmetry [4–13]. These new decay modes can serve to test the mechanism proposed in this paper. We urge the BES collaboration to carry out a systematic analysis to obtain more information on the properties of the resonant state $X(1835)$.

In the above discussions about predictions for other decay modes, only contributions from the resonance are included. There are also non-resonance effects. To isolate the resonance contributions near the resonance region, one should remove the off-resonance contributions by extrapolating the data away the resonance to the resonance region. Theoretical study of the non-resonance contributions is more complicated. We briefly discuss how the non-resonance contributions for $J/\psi \rightarrow \gamma B \bar{B}$ and $J/\psi \rightarrow \gamma \pi^+ \pi^- \eta(\eta')$ can be parameterized using $SU(3)$ flavor symmetry. The general form for non-resonance $J/\psi \rightarrow \gamma B \bar{B}$ has been discussed in [36]. As far as the $SU(3)$ structure is concerned, that is, neglecting Lorentz structure, we have the amplitudes for non-resonance contributions to $J/\psi \rightarrow \gamma B \bar{B}$ and $J/\psi \rightarrow p_1 p_2 p_3$ given by

$$M(J/\psi \rightarrow \gamma B \bar{B}) \sim \text{Tr} [\bar{B}(D_\gamma \{Q, B\} + F_\gamma [Q, B])] , \\ M(J/\psi \rightarrow \gamma p_1 p_2 p_3) \sim \tilde{a} \eta_1 \text{Tr}(QM^2) + \tilde{b} \eta_1 \eta_1 \text{Tr}(QM) + \tilde{c} \text{Tr}(QM^3), \quad (14)$$

where $Q = \text{diag}(2/3, -1/3, -1/3)$ is the electric charge matrix. D_γ , F_γ , \tilde{a} , \tilde{b} and \tilde{c} are form factors.

The complications in extracting information on the form factors D_γ and F_γ , and \tilde{a} , \tilde{b} and \tilde{c} are two-fold. One of them is that the Lorentz structure is much more complicated, that is for each of the form factors D_γ , F_γ , \tilde{a} and \tilde{b} , there are actually several of them depending on how the Dirac matrices are inserted in the $B \bar{B}$ bi-baryon product, and how derivatives are taken on the meson and baryon fields. Another is that the form factors in general depend

Table 2. $r(p_1 p_2 p_3)$ for various $G_p \rightarrow p_1 p_2 p_3$ decay modes. Here $b(\pi^+ \pi^- \eta') = |a_1|^2 - 0.376 \text{Re}(a_1 a_2^*) + 0.038 |a_2|^2$

$r(\pi^+ \pi^- \eta)$	$11.26(a_1 ^2 - 0.186 \text{Re}(a_1 a_2^*) + 0.009 a_2 ^2) / b(\pi^+ \pi^- \eta')$
$r(K^+ K^- \eta)$	$1.57 \times 10^{-3} (a_1 ^2 - 0.474 \text{Re}(a_1 a_2^*) + 0.056 a_2 ^2) / b(\pi^+ \pi^- \eta')$
$r(K^+ K^- \pi^0)$	$2.47(a_1 ^2 - 0.291 \text{Re}(a_1 a_2^*) + 0.021 a_2 ^2) / b(\pi^+ \pi^- \eta')$
$r(\eta \eta \eta)$	$0.006(a_1 ^2 - 0.538 \text{Re}(a_1 a_2^*) + 0.073 a_2 ^2) / b(\pi^+ \pi^- \eta')$

on combinations of the invariant masses of pairs of particles in the final state.

Nevertheless, fitting the data, one can obtain information on the parameters. Measurements of the branching ratios of $J/\psi \rightarrow \gamma B\bar{B}, \gamma p_1 p_2 p_3$ via non-resonance can also provide important information for understanding the whole physics picture.

In summary, we have studied some implications of a possible 0^{-+} resonance in the $\pi^+\pi^-\eta'$ spectrum in $J/\psi \rightarrow \gamma\pi^+\pi^-\eta'$ observed by BES. We have shown that it has a sizeable matrix element for $\langle 0|\tilde{G}\tilde{G}|G_p\rangle$. The branching ratios for $J/\psi \rightarrow \gamma G_p$ and $G_p \rightarrow \pi^+\pi^-\eta'$, using QCD anomaly and QCD sum rules, are determined to be $(2.61-7.37) \times 10^{-3}$ and $(2.21-10.61) \times 10^{-2}$, respectively. The coupling for the $G_p-p\bar{p}$ interaction is also determined. We conclude that a pseudoscalar 0^{-+} with large gluon content can consistently explain the data. We have also studied the branching ratios of other decay modes using $SU(3)$ flavor symmetry. We find that $J/\psi \rightarrow \gamma G_p \rightarrow \gamma(\pi^+\pi^-\eta, KK\pi^0)$ can provide useful tests for the mechanism proposed here.

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